

# The Compleat Fish Wars: Biological and Dynamic Interactions

RONALD D. FISCHER<sup>1</sup> AND LEONARD J. MIRMAN

*Department of Economics, University of Virginia, Charlottesville, Virginia 22901*

Received November 23, 1993; revised February 18, 1994

In this article we study an example of a fishery in which there are two agents, each harvesting two species of fish. There are two externalities in the model: the dynamic externality studied in Levhari and Mirman (1980) and the biological externality examined in Fischer and Mirman (1992). This model allows us to study the interactions between agents in a model in which the species of fish interact. We find the unique closed loop Nash equilibrium in the context of a differential game and we compare the resulting catch ratios to the results of the earlier papers, which allow for only one externality. We also study the efficiency of the solution. © 1996 Academic Press, Inc.

## 1. INTRODUCTION

Markets for renewable resources, particularly fishing, are characterized by many externalities. When these externalities are not internalized by a precise definition of property rights, a commons (or dynamic) externality is present. Another type of externality that arises naturally in the area of fishing models is due to the biological interactions between fish species. Both the dynamic and the biological externality have previously been analyzed separately in the context of closed loop, differential games.

Whenever the number of agents operating on the fishery is small, it is useful to model the economic situation as a differential game in order to study the effects of externalities on the decisions of the agents. In this paper we study a differential game of fishing in discrete time. The appropriate approach in these models is to find a closed loop solution to these games. The purpose of this paper is to study the interaction between the dynamic and the biological externalities in the context of a differential games.

Levhari and Mirman (LM) [1] analyzed the problem of a dynamic externality while Fischer and Mirman (FM) [2] studied the problem of a biological externality. In FM there are two countries, each fishing and consuming a different species of fish. The biological externality occurs because these two fish populations interact. The existence of externalities in these papers means that the question of efficiency occurs naturally. Although FM presents an interesting model of interacting species, the fact that each country catches and consumes only one species is not very realistic. If the species interact biologically, both countries generally have access to both species. Moreover, it seems reasonable to expect that people in each country consume both types of fish. In this paper, we assume that each country catches and consumes both species of fish. In this way, both externalities are present, i.e., the dynamic externality of LM, since both countries compete for the same fish, and the biological externality of FM, since the fish species interact. In order to study this

<sup>1</sup> This paper was written in part while R. Fischer was visiting at the University of Chile.

problem we use a differential game of fishing in discrete time. The appropriate approach in these models is to find a closed loop solution to these games and to study the efficiency of the solution.

In the case of dynamic externalities, LM have shown that overfishing occurs. While in the case of biological externalities, FM have shown that both overfishing and underfishing, depending on the type of biological interactions, can occur. In both of these papers the results are in the form of an example. This tradition is continued in this paper, since a more general analysis would require elaborate technical analysis. FM suffers from the problem that its unique Nash equilibrium closed loop solution is in dominant strategies. A dominant strategy equilibrium is one in which the optimal and therefore the equilibrium strategy for each agent is independent of the strategy of the opponent. Hence, for dominant strategies the strategy of the other agent is not taken into account when calculating the Nash equilibrium. This implies that FM does not analyze the interactions between the agents, even though the effects of the biological externality are clear. Thus it is desirable to use a model in which the strategies of the agents are not dominant in order to study how the decisions of the agents are affected by the inefficiencies created by the externality.

This paper combines the FM and LM papers in a unified model that allows us to study strategies that are not dominant when biological interactions are present. This is done in the context of an example that is consistent with the two previous models. The model also allows us to study the interactions between the dynamic and biological externalities. The closed loop solution to the differential game that results is not in dominant strategies (unlike FM). Our method is to compare the solutions to the present game to those obtained in LM and FM.

We consider three types of biological interactions: (i) symbiotic, (ii) negative, and (iii) predator-prey. These interactions are very general; for instance, a case of negative interaction would occur when there is mutual predation or if there is competition for a scarce resource. In all cases, the cooperative solution is the same as in FM.

In the case of negative interactions and symbiosis, we show that there is always overfishing (as compared to the efficient solution) when both externalities are combined, even when the biological externality by itself would have led to underfishing. This result depends on the reproductive effect of a species being stronger than the cross effect due to the other species. As in FM, we assume that the own (or direct) effect of the fish stock is stronger than the indirect effect. This assumption is used throughout the paper because it entails feasibility of the Nash equilibrium growth path. Moreover, it implies that the dynamic externality is *stronger* than the biological externality.

If we compare the extent of overfishing when both externalities are present to the case in which only the dynamic externality exists (LM), we observe that there is less overfishing when the species are symbiotic. When the species have negative interactions, there is even more more fishing than in LM when the biological interaction is present. The intuition is that agents take into account the biological interactions of one species on the other when the two externalities are present. When the interactions are negative, agents try to reduce this negative effect by overfishing as compared to LM.

Consider the case of predator-prey interactions. Comparing the case of FM to the cooperative case, there is underfishing of the predator in FM, because the

owner of the stock of the predator does not taken into account the effects on the other species' population. In this paper we show that when the agents fish both species, there is more fishing of the predator than in FM because the effect on the predator on the prey species is taken into account. The addition of the dynamic externality in this paper leads to even more fishing of the predator. On the other hand, it is not clear if in the present case more or less of the prey is caught than in FM, because there the two effects go in opposite directions; the dynamic externality leads to overfishing, while the positive biological externality on the predator leads to less fishing.

When we compare the catch ratios in the present paper to those in the cooperative case, we observe overfishing of the prey. The reason for the overfishing is that biological externality on the prey is not fully internalized by the agents whereas the dynamic externality always lead to overfishing. On the other hand, these effects work in opposite directions for the case of the predator, so it is not clear if there is overfishing or underfishing (as compared to cooperation). Which of the two obtains will depend on the parameters of the biological interaction and on the intertemporal discount rates.

## 2. GENERAL PROBLEM

Suppose there are two agents (the owners), each of whom can fish two fish species. Let  $(x_t, y_t)$  be the stocks of each type of fish at time  $t$ . The fish species interact biologically. The general biological growth rule for the fish is given by

$$\begin{aligned} x_{t+1} &= f(x_t, y_t) \\ y_{t+1} &= g(x_t, y_t). \end{aligned} \tag{1}$$

Given the total catch of a species,  $c_{it} \equiv c_{1it} + c_{2it}$ ;  $i = 1, 2$  each period's population is described by

$$\begin{aligned} x_{t+1} &= f((x_t - c_{1t}), (y_t - c_{2t})) \\ y_{t+1} &= g((x_t - c_{1t}), (y_t - c_{2t})). \end{aligned} \tag{2}$$

Owners derive utility from consuming both species of fish,  $u(c_{i1t}, c_{i2t})$ . Both owners are interested in maximizing the sum of discounted utility. The first owner solves ( $0 < \delta_1 < 1$ ),

$$\text{Max}_{\{c_{11t}, c_{12t}\}} \sum_{t=1}^{\infty} \delta_1^t u(c_{11t}, c_{12t})$$

subject to

$$\begin{aligned} x_t &\geq c_{1t} \geq 0, \quad y_t \geq c_{2t} \geq 0 \\ x_{t+1} &= f((x_t - c_{1t}), (y_t - c_{2t})) \\ y_{t+1} &= g((x_t - c_{1t}), (y_t - c_{2t})), \end{aligned} \tag{3}$$

and similarly for owner 2. In this general form, the properties of the solution to these problems is unknown. Here we study a particular specification which can be

solved for a simple closed form solution. We can then compare the results to those that obtain when there is only a single externality. We believe the properties of the solutions obtained for this example give us insight into the properties of the solutions to problems of the general form described in (3).

### 3. THE MAIN EXAMPLE

Let the biological growth rule be given by

$$\begin{aligned} x_{t+1} &= x_t^{\alpha_1} y_t^{\beta_1} \\ y_{t+1} &= x_t^{\beta_2} y_t^{\alpha_2}. \end{aligned} \tag{4}$$

The utility function of each owner as a function of the catch  $c_{ij}$ ,  $i, j = 1, 2$  is

$$u(c_{i1}, c_{i2}) = \nu_1 \log c_{i1} + \nu_2 \log c_{i2}, \tag{5}$$

where the  $\nu_k$  indicates the relative weight given by the agents to consumption of either species of fish.<sup>2</sup>

The intertemporal rate of substitution is given by

$$0 < \delta_1, \delta_2 < 1$$

The  $\alpha_i > 0$  represent the direct (or own) effect of the stock of a fish species on the next period's stock of that species. The  $\beta_i$  represent the indirect effect of the stock of the other species on the stock of the species being considered. We assume that  $\alpha_i > \beta_i$ , i.e., the reproductive effect of the own species is stronger than the indirect effect due to the other species. There are three combinations of the  $\alpha_i$ 's and  $\beta_i$ 's that have attractive interpretations.

i. When both  $\beta_i$  are positive, the interaction between the species corresponds to a symbiotic relation.

ii. When both  $\beta_i$  are negative, the relation between two species is negative, as when species compete for a resource or are mutual predators.

iii. Finally, the case when the  $\beta_i$ 's have opposite signs corresponds to the case of predator-prey interactions.<sup>3</sup>

### 4. THE NONCOOPERATIVE EQUILIBRIUM

In order to determine the solutions to the agent's problems we follow FM (1992) in assuming the existence of value functions corresponding to a Nash equilibrium.

<sup>2</sup> When there are two subindexes the first refers to the agent and the second to the fish species.

<sup>3</sup> See Mirman and Fischer (1992) for the dynamics of this model. There it is shown that the steady state is stable if (i)  $\alpha_i < 1$  and either  $\alpha_i + |\beta_i| < 1$  (when  $\text{sgn}(\beta_1) = \text{sgn}(\beta_2)$ ) or  $\alpha^2 + \beta^2 < 1$  (when  $\text{sgn}(\beta_1) = -\text{sgn}(\beta_2)$ ).

Let these functions be  $V_i(x, y)$ ,  $i = 1, 2$ . By Bellman's principle, these functions must satisfy

$$V_1(x, y) = \text{Max} \left\{ \nu_1 \log c_{11} + \nu_2 \log c_{12} \right. \\ \left. + \delta_1 V_1 \left[ (x - c_1)^{\alpha_1} (y - c_2)^{\beta_1}, (x - c_1)^{\beta_2} (y - c_2)^{\alpha_2} \right] \right\}$$

s.t.

$$0 \leq c_{11} + c_{21} \equiv c_1 < x, \quad 0 \leq c_{12} + c_{22} \equiv c_2 < y, \quad (6)$$

and similarly for agent 2. To obtain the noncooperative solution, we assume that it has the functional form

$$V_i(x, y) = A_i \log x + B_i \log y + D_i, \quad i = 1, 2, \quad (7)$$

where  $A_i$ ,  $B_i$ , and  $D_i$  are constants that depend on the parameters of the model. Suppose that the catch ratios are linear in stocks

$$c_{i1} = \gamma_{i1}^{\text{NC}} x, \quad c_{i2} = \gamma_{i2}^{\text{NC}} y, \quad i = 1, 2, \quad (8)$$

where NC indicates that these catch ratios are noncooperative.<sup>4</sup> Then,

$$x - c_1 = x(1 - \gamma_{11} - \gamma_{21}) \\ y - c_2 = y(1 - \gamma_{12} - \gamma_{22}). \quad (9)$$

Identifying terms in (6) using (7), (8), and (9) we have

$$A_1 = \delta_1 \alpha_1 A_1 + \delta_1 \beta_2 B_1 + \nu_1 \\ B_1 = \delta_1 \beta_1 A_1 + \delta_1 \alpha_2 B_1 + \nu_2. \quad (10)$$

These equations can be solved to obtain  $A_i$ ,  $B_i$  in terms of the fundamental parameters. Replacing the expressions obtained for  $A_i$  and  $B_i$  into (7) and using it to maximize the r.h.s. of (6), we obtain expressions for the catch ratios in terms of the utility and biological parameters (see Appendix).

The general solution is not very instructive,<sup>5</sup> so we consider simpler cases, which can be related to those that have been studied in the literature, such as LM and FM. In the analysis that follows, we assume  $\nu_1 = \nu_2 = 1$ ,  $\alpha_1 = \alpha_2$ ,  $|\beta_1| = |\beta_2|$  and  $\delta_1 = \delta_2 = \delta$ .<sup>6</sup>

<sup>4</sup> The cooperative case of this model is identical to the cooperative case in Mirman and Fischer.

<sup>5</sup> For the case where  $\delta_1 = \delta_2 = \delta$ ,  $\nu_1 = \nu_2 = 1$  we have

$$\gamma_{11}^{\text{NC}} = \frac{(1 - \alpha_1 \delta)(1 - \alpha_2 \delta) - \delta^2 \beta_1 \beta_2}{(1 - \alpha_2 \delta)(2 - \alpha_1 \delta) + \beta_2 \delta(1 - \beta_1 \delta)} = \gamma_{21}^{\text{NC}}.$$

A similar expression, where the subindexes are interchanged everywhere, holds for the catch ratio of the other species.

<sup>6</sup> When  $\alpha_1 = \alpha_2$ ,  $\beta_1 = \beta_2$ , the case  $\delta_1 \neq \delta_2$  can be analyzed. We get

$$\gamma_{11} = \frac{\delta_2(1 - \delta_1(\alpha + \beta))}{\delta_1(1 - \delta_2 \alpha) + \delta_2(1 - \delta_1 \beta)} = \gamma_{12} < 1$$

and a similar expression (reversing the places of  $\delta_1$  and  $\delta_2$  in the numerator) for  $\gamma_{12} = \gamma_{22}$ . Suppose that starting with  $\delta_1 = \delta_2$  we increase the impatience of the first agent. His catch ratio increases while the other agent's catch ratio falls. The total effect is an increase in the global catch ratio:  $\partial \gamma_{11} / \partial \delta_1 = -\delta_2 / \Psi^2$ ,  $\partial \gamma_{21} / \partial \delta_1 = \delta_2(1 - \delta_2(\alpha + \beta)) / \Psi^2$ , where  $\Psi \equiv \delta_1(1 - \delta_2 \alpha) + \delta_2(1 - \delta_1 \beta)$ .

TABLE I  
Catch Ratios (Case  $\beta_1 = \beta_2$ )

Levhari and Mirman $\frac{1 - \alpha\delta}{2 - \alpha\delta}$ Dynamic externality		Levhari and Mirman $\frac{1 - \alpha\delta}{2}$ Cooperation
	Present case $\frac{1 - \alpha\delta - \beta\delta}{2 - \alpha\delta - \beta\delta}$ Dynamic + biological externality	
Fischer and Mirman $\frac{(1 - \alpha\delta)^2 - \beta^2\delta^2}{2(1 - \alpha\delta)}$ Biological externality		Fischer and Mirman $\frac{1 - \alpha\delta - \beta\delta}{2}$ Cooperation

*Note.* The catch ratios for the case of the biological externality are one half the catch ratios in FM. The reason is that now each agent harvests the two species, so to make the catch ratios of each species comparable we must divide the catch ratios by two.

The first case to be considered is the one in which  $\beta_1 = \beta_2$ , corresponding to symbiosis or to negative interactions. Table I presents the catch ratios in each model. Observe that the cooperative case, when both externalities are present, is identical to the cooperative case of FM.

The expressions shown in Table I correspond to the catch ratios for each species by each owner. The first thing to notice is that feasibility (positive catch ratios, i.e.,  $\alpha > |\beta|$ ) ensures that under noncooperation there is always overfishing as compared to the cooperative solution. In contrast, in FM there is underfishing as compared to cooperation when the interactions are negative. Hence the dynamic externality is "stronger," because when both externalities are combined, overfishing always results. Intuitively, the need to compete for the resource overwhelms the externality due to the biological interactions between the species. Nevertheless, the negative biological interaction mitigates the extent of overfishing, because the mutual predation between the species is partially internalized by the agents. When the species are symbiotic, the rate of overfishing above the cooperative solution is exacerbated by the biological interaction. The reason is that in this case, the biological externality also leads to overfishing.

We now compare the catch ratios when only the dynamic externality is present (LM) to the situation when both externalities exist. When we compare the catch ratios in LM to those occurring when the two externalities are present, the difference in the catch ratios depends on the sign of  $\beta$ . When the species are symbiotic, the dynamic externality by itself (LM) leads to a higher catch ratio than when the two externalities are combined. The intuition for this result is simple; when the fish species are symbiotic, the agents take this into account when they decide how much they should fish of each species. This leads to less fishing (even though still nonoptimal) than when there are two independent fish populations (the LM case). On the other hand, when there is a negative interaction (mutual predation) between species, the agents take this into account and cull more from

their stocks than would agents who face no biological externality. Thus the catch ratios with the two externalities are higher than in LM.

Consider now the relationship between the catch ratios in FM, when only the biological externality is present and those of the present case, when both externalities are present. The catch ratio in FM is

$$\gamma^{\text{FM}} \equiv \frac{(1 - \alpha\delta)^2 - \beta^2\delta^2}{2(1 - \alpha\delta)} = \frac{(1 - \alpha\delta - \beta\delta)}{(2 - \alpha\delta - \beta\delta)} \frac{(1 - \alpha\delta + \beta\delta)(2 - \alpha\delta - \beta\delta)}{2(1 - \alpha\delta)}. \quad (11)$$

Notice that the the catch ratio in FM is smaller than the catch ratio with both externalities if

$$\frac{(1 - \alpha\delta + \beta\delta)(2 - \alpha\delta - \beta\delta)}{2(1 - \alpha\delta)} < 1. \quad (12)$$

This is equivalent to

$$(\alpha - \beta)(1 - \delta(\alpha + \beta)) > 0,$$

which is always true by the feasibility conditions. Therefore, the catch ratio is always higher when the two externalities are present. In particular, observe that if  $\beta < 0$ , there is underfishing when only the biological externality is present, but if both externalities are present, there is overfishing. This result indicates that the dynamic externality is “stronger” than the biological externality.<sup>7</sup>

The remaining case is the predator–prey interaction. Here,  $\alpha_1 = \alpha_2 = \alpha$ ,  $\delta_1 = \delta_2$ ,  $\beta_1 = -\beta_2 = \beta > 0$ , so the second species is the prey. Recall that

$$\Delta = (1 - \delta_2\alpha_2)(1 - \delta_2\alpha_1) - \delta_2^2\beta_1\beta_2 = (1 - \alpha\delta)^2 + \beta^2\delta^2.$$

Solving for the catch ratio of the predator,

$$\gamma_{11} = \gamma_{21} = \Delta/(\Delta + 1 - \delta(\alpha + \beta)) \quad (13)$$

while for the prey

$$\gamma_{12} = \gamma_{22} = \Delta/(\Delta + 1 - \delta(\alpha - \beta)). \quad (14)$$

Comparing this expression with the one that obtains under cooperation (see FM), which is

$$\gamma_2^C = \Delta/(1 - \delta(\alpha - \beta)), \quad (15)$$

we have  $\gamma_{12}^{\text{NC}} + \gamma_{22}^{\text{NC}} > \gamma_2^C$ , since  $1 - \delta(\alpha - \beta) > \Delta$ . Hence, in the noncooperative case there is overfishing of the prey as compared to the cooperative case. On the other hand, depending on the values of the parameters, there will be over- or underfishing of the predator.

<sup>7</sup> This fact reflects our assumption that the own effects of fish stocks are larger than the cross effects.

Finally, the catch ratios in FM and when there are two externalities are compared in the noncooperative case. In FM,

$$\gamma_i^{\text{FM}} = \Delta / (1 - \alpha\delta), \quad i = 1, 2. \quad (16)$$

Consider the difference in global catch ratios for the predator,

$$2\gamma_{11} - \gamma_i^{\text{FM}} = 2\Delta / (\Delta + (1 - \delta(\alpha + \beta))) - \Delta / (1 - \alpha\delta),$$

where  $\text{sgn}(2\gamma_{11} - \gamma_i^{\text{FM}}) = 1 - \delta(\alpha - \beta) - \Delta > 0$ .<sup>8</sup> Thus, there is always more fishing of the predator when the two externalities are present, given that (i) the dynamic externality added to the biological externality leads to more fishing and (ii) each agent takes into account the effect fishing the predator has on the population of the prey. For the prey, the results are ambiguous, since  $\text{sgn}(2\gamma_{21} - \gamma_i^{\text{MF}}) > (<) 0$ , reflecting the fact that the two effects work at cross purposes.<sup>9</sup>

## APPENDIX

Solving Eqs. (10) results in

$$\begin{aligned} A_1 &= \{\nu_1(1 - \delta_1\alpha_2) + \nu_2\delta_1\beta_2\}/\Delta \\ B_1 &= \{\nu_1\delta_1\beta_1 + \nu_2(1 - \delta_1\alpha_1)\}/\Delta \\ \Delta &= (1 - \delta_1\alpha_2)(1 - \delta_1\alpha_1) - \delta_1^2\beta_1\beta_2. \end{aligned} \quad (A1)$$

Similarly, for agent 2 we have

$$\begin{aligned} A_2 &= \{\nu_1(1 - \delta_2\alpha_2) + \nu_2\delta_2\beta_2\}/\Delta \\ B_2 &= \{\nu_1\delta_2\beta_1 + \nu_2(1 - \delta_2\alpha_1)\}/\Delta \\ \Delta &= (1 - \delta_2\alpha_2)(1 - \delta_2\alpha_1) - \delta_2^2\beta_1\beta_2. \end{aligned} \quad (A2)$$

Inserting (7), (8), and (9) into (4) and maximizing, we get

$$\begin{aligned} \frac{\nu_1}{c_{11}} &= \frac{\delta_1 A_1 \delta_1 + \delta_1 B_1 \beta_2}{(x - c_{11} - c_{12})} \\ \frac{\nu_2}{c_{21}} &= \frac{\delta_1 A_1 \beta_1 + \delta_1 B_1 \alpha_2}{(y - c_{21} - c_{22})} \\ \frac{\nu_1}{c_{12}} &= \frac{\delta_2 A_2 \alpha_1 + \delta_2 B_2 \beta_2}{(x - c_{11} - c_{12})} \\ \frac{\nu_2}{c_{22}} &= \frac{\delta_2 A_2 \beta_1 + \delta_2 B_2 \alpha_2}{(y - c_{21} - c_{22})} \end{aligned} \quad (A3)$$

Substituting (A1) and (A2) into (A3), we obtain the catch ratios in terms of the parameters of consumption and production.<sup>10</sup>

<sup>8</sup> Note that  $\Delta = (1 - \alpha\delta)^2 + \beta^2\delta^2 < 1 - \delta(\alpha - \beta)$  because  $(1 - \alpha\delta)^2 < (1 - \alpha\delta)$  and  $\beta^2\delta^2 < \beta\delta$ .

<sup>9</sup>  $\text{sgn}(2\gamma_{12} - \gamma_i^{\text{MF}}) = \text{sgn}(1 - \delta(\alpha + \beta) - \Delta) > (<) 0$ .

<sup>10</sup> Note that the first and third equations can be solved for the catch ratios independently of the other two equations.



## REFERENCES

1. Ronald D. Fischer and Leonard J. Mirman, Strategic dynamic interactions: Fish wars, *J. Econom. Dynam. Control* **16**, 267–287 (1992).
2. David Levhari and Leonard J. Mirman, The great fish war: An example using a dynamic Cournot–Nash Solution, *Bell J. Econom.* **11**(1), 322–334 (1980).